School of Isolated and Distance Education
MATHEMATICS SPECIALIST Year 11

Test 3 2023

Section 1: Calculator Free

Time allowed for this section

Working time: 20 minutes

Mark allocation: 29 marks

PERMISSIBLE ITEMS

Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special Items: none

STANDARD FORMULAE SHEET IS PROVIDED

NO OTHER ITEMS MAY BE TAKEN INTO THE EXAMINATION ROOM

INSTRUCTIONS FOR CANDIDATES

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

All work must be done in the space provided. Should you need extra working area you may use the blank pages at the end.

Student's name: Solutions

SIDE Teacher's name: ______________________

SUPERVISOR'S DECLARATION

I declare that this test paper has been completed without assistance by the student named above. The time and resource restrictions have been observed and the student has NOT accessed additional notes other than the one A4 page allowed, texts, reference books, the internet, a computer, a mobile phone or other electronic device. I understand that this paper will not be counted for assessment if these conditions have not been met and that notifications will occur.

Supervisor's name: ______________________

Supervisor's signature: ______________ **Date:** __________

QUESTION 1 **[1, 2, 3, 4 = 10 marks]**

(a) Evaluate $\frac{20!}{18!2!}$ $= \frac{20 \times 19}{2}$

$= 190$ ✓

(b) Simplify $\frac{n!}{(n-2)!}$ $= \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$ ✓

$= n(n-1)$ ✓

(c) Find the value of n, if $^{n}C_{2} = 3$

$3 = \frac{n!}{(n-2)!2!}$

$3 = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2}$ ✓

$0 = n^2 - n - 6$

$0 = (n-3)(n+2)$ ✓

$n = 3$ $\quad n = -2$ ← Reject this → ✓ $\therefore n = 3$

Full mark if they use trial and error

(d) Solve for n, if $^{n}P_{4} : {}^{n}C_{2} = 12 : 1$

$\frac{n!}{(n-4)!} \div \frac{n!}{(n-2)!\,2!} = \frac{12}{1}$ ✓ each step

$\frac{\cancel{n!}}{(n-4)!} \times \frac{(n-2)!\,2!}{\cancel{n!}} = 12$

$\frac{2(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}} = 12$ ✓ for each line

$(n-2)(n-3) = 6$

$n^2 - 5n + 6 - 6 = 0$ ✓

$n(n-5) = 0$

$n = 0$ or $n = 5$ ✓

−1 If the answers ~~are~~ included $n = 0$.

QUESTION 2 [2, 3 = 5 marks]

The digital sum of a natural number is defined to be the sum of its digits. For example, the digital sum of 123 is 1 + 2 +3 = 6

(a) Nineteen two-digit numbers are selected. Prove that at least two of them have the same digital sum

✓ for 18 There are 18 possible digital sums

✓ for conclusion If there are 19 numbers, there is some digital sum occurs at least twice

(b) Supposed that 82 three digital numbers are selected. Prove that at least four of them have the same digital sum.

There are 27 ✓ possiple digital sums

If there are 82 numbers

✓ $82 = 27 \times 3 + 1$. There is some digital sum that occurs at least 4 times. ✓

✓ for 27

✓ for $82 = 27 \times 3 + 1$

✓ for conclusion.

QUESTION 3 [1, 3 = 4 marks]

By definition, the k^{th} term in row n of Pascal's Triangle, given as $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

(a) With $n = 5$ and $k = 2$, show that the left and right sides of the identity are equal.

✓ for both sides

$LHS = {}^5C_2 = 10$

$RHS = {}^4C_1 + {}^4C_2 = 10$ ✓ $\Rightarrow LHS = RHS.$

(b) Prove the identity is always true, subject to restrictions $n, k \in Integers, n \geq k \geq 1.$

$$RHS = \binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-1-k)!}$$

✓ $= (n-1)!\left(\frac{1}{(k-1)!(n-k)(n-k-1)!} + \frac{1}{k(k-1)!(n-1-k)!}\right)$

for each line

$=(n-1)!\left(\frac{k + n-k}{(k-1)!(n-k)(n-k-1)!\,k}\right)$

✓ $= (n-k)!\left(\frac{n}{k!(k-1)!}\right)$

✓ $= \frac{n!}{k!(n-k)!} = \binom{n}{k} = LHS$

QUESTION 4 [1, 1, 1, 3 = 6]

Use Pascal's triangle provided at the end of this test to answer the following.

How many groups of six can be chosen from ~~four~~ 5 women and four men at a workplace given:

(a) There is no restriction on who is in the group

$${}^{9}C_6 = 84 \checkmark$$

Full mark if the formulas are correct.

(b) The workplace needs four women and ~~one~~ 2 man in the group

$${}^{5}C_4 \, {}^{4}C_2 = 5 \times 6 = 30 \checkmark$$

(c) The group contains at most one man?

$${}^{4}C_1 \, {}^{5}C_5 = 1 \times 4 = 4 \checkmark$$

(d) The group contains at least one woman? ✓

$$\checkmark \; \underbrace{{}^{5}C_2 \, {}^{4}C_4 + {}^{5}C_3 \, {}^{4}C_3}\, + \underbrace{{}^{5}C_4 \, {}^{4}C_2 + {}^{5}C_5 \, {}^{4}C_1}$$

$$= 10 + 40 + 30 + 4 = \overset{84}{\cancel{64}} \checkmark$$

QUESTION 5 [2, 2 = 4 marks)

A line of Pascal's triangle is shown below.

1 9 36 84 126 126 84 36 9 1

With reference to that line, demonstrate the truth (or otherwise) of these general statements.

(a) (i) ${}^{n}\mathbf{C}_{r} = {}^{n}\mathbf{C}_{n-r}$ $n = 9$ $r = 2$

$$\left.\begin{array}{l} LHS = {}^{9}C_2 = 36 \\ RHS = {}^{9}C_7 = 36 \end{array}\right] \checkmark = LHS$$

True. ✓

(ii) ${}^{n}\mathbf{C}_{r} = 2\,{}^{n}\mathbf{C}_{n-r}$

$${}^{9}C_2 = 2 \; {}^{9}C_7 \checkmark$$

$$36 \neq 2 \times 36$$

False ✓

Do not have to be done algebraically!

End of Calculator Free section.

Pascal's Triangle

Value of n

0 $\binom{0}{0}$

1 $\binom{1}{0}$ $\binom{1}{1}$

2 $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

3 $\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$

4 $\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$

$\longrightarrow$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1

School of Isolated and Distance Education
MATHEMATICS SPECIALIST Year 11

Test 3 2023

Section 2: Calculator assumed

Time allowed for this section

Working time: 30 minutes

Mark allocation: 39 marks

PERMISSIBLE ITEMS

Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special Items: Formulae Sheet, CAS calculator, ONE A4 page of notes

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Supervisor's signature:________________ **Date:**__________

QUESTION 6 **[1, 2, 4 = 7 marks]**

(a) How many 5-character passwords can be created from the lower-case letters of the alphabet without repetition?

$$^{26}P_5 = 7\,893\,600 \checkmark$$

(b) How many 5-character passwords can be created from the lower-case letters of the alphabet, without repetition, that contain exactly three vowels?

$$^5C_3 \; ^{21}C_2 \times 5! = 252000 \checkmark$$

(c) How many 8-character passwords can be created from the lower-case letters of the alphabet and the digits 0 to 9, without repetition, that contain exactly three vowels OR exactly three consonants?

(4 marks)

$$n(3V) = \binom{5}{3}\binom{31}{5}\,8! \checkmark$$

$$n(3C) = \binom{21}{3}\binom{15}{5}\,8! \checkmark$$

$$n(3V \cap 3C) = \binom{5}{3}\binom{21}{3}\binom{10}{2} \times 8! \checkmark$$

$$\begin{aligned} n(3V \cup 3C) &= n(3V) + n(3C) - n(3V \cap 3C) \\ &= 8!\left[\binom{5}{3}\binom{31}{5} + \binom{21}{3}\binom{15}{5} - \binom{5}{3}\binom{21}{3}\binom{10}{2}\right] \\ &= 8! \times 5094600 \checkmark \end{aligned}$$

Full mark if they have the combinations and permutation correct.

QUESTION 7 **[~~1~~, ~~2~~, 2 = ~~5~~ marks]** 2 2 6

How many ways can two blue, three black and four green marbles to be arranged in a row:

a) Without restriction

$$\frac{9!}{2!\,3!\,4!} \checkmark = 1260 \checkmark$$

b) If the first and the last flags are blue

$$\frac{7!}{3! \times 4!} = 35 \checkmark\checkmark \quad \cancel{8! \times 2!} = \cancel{80640} \checkmark\checkmark$$

c) If three black are adjacent?

$$\frac{7!}{2! \times 4!} \quad \cancel{7! \times 3!} = \cancel{30240} \checkmark\checkmark \quad 105 \checkmark\checkmark$$

Full mark if they have the formulas correct

QUESTION 8 **[6, 1, 2, ~~3~~ 2 = ~~12~~ 11 marks]**

(a) A box contains 400 balls, each of which is blue, red, green, yellow or orange. The ratio of blue to red to green balls is 1 : 4 : 2. The ratio of green to yellow to orange balls is 1 : 3 : 6. What is the smallest number of balls that must be drawn to ensure that at least 50 balls of one colour are selected?

B : R : G G : Y : O
1 : 4 : 2 1 : 3 : 6
B R G Y O
∴ ✓ 1 : 4 : 2 : 6 : 12 = 25 ✓

$B: 400 \times \frac{1}{25} = 16$
$R: 400 \times \frac{4}{25} = 64$ ✓
$Y: 400 \times \frac{6}{25} = 96$
$O: 400 \times \frac{12}{25} = 192$ ✓ Follow through if they get the ratio wrong.
$G: 400 \times \frac{2}{25} = 32$

5 holes : 16B 32G 49R 49Y 49O
Total $16 + 32 + 3 \times 49 + 1$ ✓ $= 196$ balls ✓

(b) Consider the letters in the word CULLACAABARDEE, an Aboriginal Noongar word meaning meeting place. Determine the number of different:

(i) Combinations of 4 letters chosen from the consonants in the word.

5 consonants {C, L, D, R, D} $\binom{5}{4} = 5$ ✓

(ii) How many ways can we arrange this word?

C − 2
L − 2
A − 4
E − 2

$\frac{14!}{2!\,2!\,2!\,4!}$ ✓ $= 454\,053\,600$ ✓

(iii) How many subsets of at least 1 letter can be formed from the word KAYA?

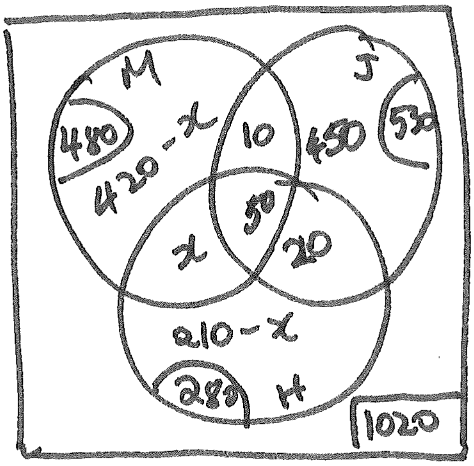
$2^4 - 1$ ✓ $= 15$ ✓ Full mark if they have formulas correct

QUESTION 9 **[6, 1, 1 = 8 marks]**

In a group of 1020 students:

- 810 studied exactly one of Mathematics, Japanese and Health.
- The number of students studying Health is 200 less than the number of students studying Mathematics and 250 less than the number of students studying Japanese.
- 60 students study Mathematics and Japanese and of these 10 do not study Health.
- One quarter of Health students also study Japanese and 20 of these do not study Mathematics.

(a) How many students study Mathematics only?



$n(H) = 280$ ✓

$n(M) = 480$ ✓

$n(J) = 530$ ✓

$(210 - x) + (420 - x) + 450 = 810$ ✓

$2x = 170$

$x = 135$

n (Math only) = 420 − 135 = 285 ✓

Follow through mark here if they have wrong x.

(b) How many students study none of these subjects?

n(none) = 1030 − 530 − 210 − 285

= 5 ✓

(c) How many students study exactly two subjects?

10 + 20 + 135 = 165 ✓

QUESTION 10 **[1, ~~2~~ 1, 3, 1, 1 = ~~8~~ 7 marks]**

Consider the set of integers 1 and 96 inclusive. Let sets A and B consists of thos integers that are multiples of 6 and 8 respectively.

(a) What is the lowest common multiple of 6 and 8?

24 ✓

(b) How many integers belong $A \cap B$? ?

$n(A \cap B) = 96 \div 24 = 4$ ✓

or $\{24, 48, 72, 96\}$ or

(c) How many integers are divisible by 6 or 8?

$n(A) = 96 \div 6 = 16$ ✓✓

$n(B) = 96 \div 8 = 12$

$n(A \cap B) = 4$ (from part b.)

$n(A \cup B) = 16 - 12 - 4 = 24$ ✓

Follow throug if they get part (b) wrong

(d) How many integers that are not divisible by 6 and 8?

$96 - 24 = 72$ ✓

-> THIS IS WRONG <-
Correct answer:
96 - 4 = 92 (1 Mark)

(e) An integer is chosen at random, what is the probability that it is not divisible by 6 or 8?

$$\frac{72}{96} = \frac{3}{4}$$ ✓ simplified

Follow through here. if they get part (d) wrong.

End of Test